

Entanglement of nanoelectromechanical oscillators by Cooper-pair tunneling

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We demonstrate that entanglement of two macroscopic nanoelectromechanical resonators – coupled to each other via a common detector, a tunnel junction – can be generated by running a current through the device. This can be most efficiently achieved if the two oscillators are initially both prepared in their ground states. We propose two kinds of setups where the generation of entanglement can be realized by two different means. In the first setup, the oscillators are indirectly coupled via common fermionic reservoirs with long coherence times. While this setup gives valuable insight in the physics of this open quantum system, the second proposed setup, an Andreev entangler, represents a novel and feasible way of entangling two nanomechanical oscillators. In the Andreev entangler, a split Cooper-pair that coherently tunnels to each oscillator, mediates their coupling and thereby generates entanglement between them.

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Introduction. — Since the early days of quantum mechanics, entanglement has been one of its most fascinating key elements [1]. The study of entanglement not only is concerned with fundamental questions but also with applications, for instance, entangled qubits being the building blocks for quantum computers. Two of the main questions when studying entanglement are: How can one generate entanglement in a multipartite system and what is a good measure for entangled states? For discrete variable multipartite systems with, for instance, qubits as representatives, the former two questions have been extensively studied theoretically as well as experimentally [2–4].

Only within the last few years, nanoelectromechanical systems (NEMS) have been providing an experimental and theoretical playground for studying quantum effects on a macroscopic length scale. Among the latest experimental achievements are cooling a nanomechanical resonator to its ground state [5–7] and engineering of non-trivial quantum mechanical states [8]. On the theoretical side, entanglement generation in NEMS has been a goal for many years. In the pioneering work of Ref. [9], a route towards entanglement in NEMS was proposed making use of a global non-adiabatic change of the interaction strength in a one-dimensional chain of nanomechanical oscillators, leading to entanglement of two spatially well separated oscillators of the chain. This type of entanglement has, however, up to date not been realized in experiments. Subsequent proposals in (hybrid) nanooptomechanical systems have suggested to generate entanglement between a moveable mirror of a cavity and a collective spin ensemble of an atomic medium [10], a Bose-Einstein condensate [11, 12], the cavity field itself [13], or another mirror [14–16].

Entanglement creation and verification of quantum systems, as for instance, atoms and photons is a theoretically well understood subject and routinely done experimentally in various labs [17–19]. Entanglement of quan-

tum systems with constituents being more than just one atom or photon, as for instance, collective spin ensembles and Josephson qubits, have also been studied recently [20, 21]. These kinds of systems are often times discrete variable systems with a finite dimensional Hilbert space and true quantum systems in the sense that they do not have any classical counterpart. With the laws of quantum mechanics describing atomic and subatomic systems and the laws of classical Newtonian mechanics describing macroscopic objects, there must exist a boundary between the quantum and the classical world. This quantum to classical boundary is still subject in contemporary physics [22, 23]. Emerging questions are: What causes the quantum to classical transition? Where is the limit that causes objects to behave quantum mechanically and not classically? How macroscopic can an object be to be still considered quantum?

This work is motivated by exactly these questions. Our aim is to investigate the quantum mechanical feature per se, entanglement, in a quantum harmonic oscillator that behaves as classically as it gets (in the sense of the Ehrenfest theorem). We like to study the challenge to generate entanglement between two nanomechanical oscillators, being representatives of two mesoscopic continuous variable systems. To this end, we propose two fundamentally new schemes towards entanglement generation in NEMS.

The first scheme, Setup (A), consists of two nanoelectromechanical oscillators coupled to a common measurement device/reservoir and is depicted in Fig. 1(a). This setup being rather academic serves us as a model and provides us with new insight into the situation where two nanomechanical oscillators are effectively coupled through two common fermionic baths. This is a less studied case compared to the more established case of coupling two oscillators to a common bosonic bath, for instance, within a Caldeira-Leggett model [24]. For a more feasible version of Setup (A), we introduce a second

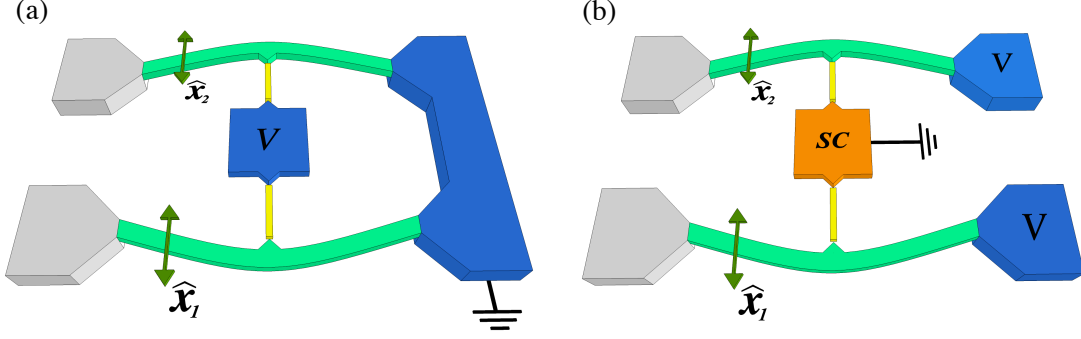


FIG. 1. (Color online) (a) Schematic setup of a bipartite continuous variable quantum system, realized as two nanoelectromechanical oscillators (green) in a tunnel junction setup which share common fermionic reservoirs (blue). Yellow lines indicate tunnel junctions that are needed for the generation of entanglement. (b) Two nanomechanical oscillators are effectively coupled in an Andreev entangler setup due to a Cooper-pair that is split. Each of the two electrons of the Cooper-pair is assumed to tunnel from the superconductor (orange) onto a different oscillator.

system, Setup (B), where we bypass the common (right) reservoir. Setup (B) is inspired by an Andreev entangler setup [25] and schematically depicted in Fig. 1(b).

Method.— Both of our proposed setups are best analyzed in a system-bath approach. The total system is described by the generic Hamiltonian

$$H = H_S + H_B + H_{SB}, \quad (1)$$

where H_S and H_B describe the system and bath degrees of freedom, respectively. The system Hamiltonian for Setup (A) and (B) is given by $H_S = \sum_{i=1,2} \hat{p}_i^2/2m_i + m_i\Omega_i^2\hat{x}_i^2/2$ which corresponds to two nanomechanical oscillators with effective mass m_i , frequency Ω_i , and position and momentum operators \hat{x}_i and \hat{p}_i , respectively. For simplicity, we later assume two identical oscillators, i.e., $\Omega_1 = \Omega_2 = \Omega$, and $m_1 = m_2 = m$. The coupling between the system and the bath is given by H_{SB} which we will specify below for Setup (A) and (B) separately. Our goal is to investigate the time evolution of entanglement between two nanomechanical oscillators. A Gaussian bipartite system is fully characterized by its covariance matrix $\sigma_{ij}(t) = \text{Tr}_S[\rho_S(t)\{\xi_i, \xi_j\}/2]$, where $\rho_S(t) = \text{Tr}_B[\rho(t)]$ is the system's density matrix and $\vec{\xi} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)^T$. As a measure for entanglement, we use the logarithmic negativity E_N [27]. The logarithmic negativity is directly related to $\tilde{\sigma}(t) = \sigma^{T_1}(t)$ through its smallest symplectic eigenvalue $\tilde{\lambda}(t)$ and given by $E_N(t) = \max\{0, -\log_2[2\tilde{\lambda}(t)]\}$, where $\tilde{\sigma}(t)$ is the partially transposed (with respect to subsystem 1) covariance matrix [28]. We calculate the time dependence of the entries of $\sigma(t)$ with an equation of motion for the system's density matrix $\rho_S(t)$, where we treat the non-Markovian regime by employing a time convolutionless master equation method [29]. The non-Markovian dynamics implies that the system's time evolution depends on its history, reflected in the equation of motion by time dependent damping and decoherence kernels. In

the Schrödinger picture, the master equation in the Born approximation reads (we put $\hbar = 1$)

$$\begin{aligned} \dot{\rho}_S(t) &= -i[H_S, \rho_S(t)] \\ &- \int_0^t d\tau \text{Tr}_B[H_{SB}, [H_{SB}(\tau - t), \rho_S(t) \otimes \rho_B]] . \end{aligned} \quad (2)$$

With Eq. (2), expectation values of arbitrary system operators X (or products of them) can be calculated by solving $d/dt\langle X(t) \rangle = \text{Tr}_S[\dot{\rho}_S(t)X]$.

Setup A.— The first setup we propose, can be seen as a natural extension of the experimental realization of one oscillator coupled to a single atomic point contact [30]. Setup (A) consists of two nanoelectromechanical oscillators that are both coupled via an atomic point contact to common electronic reservoirs, cf. Fig. 1(a). Importantly, the oscillators are not directly coupled to each other (as e.g. in Ref. [26]), but only through the electron reservoirs which act as fermionic baths. The yellow tunnel junctions in Fig. 1(a) are sufficient to generate an entangled state of the two oscillators. The Hamiltonian for Setup (A) is of the form of Eq. (1), where $H_B = \sum_r \epsilon_r \psi_r^\dagger \psi_r + \sum_l \epsilon_l \psi_l^\dagger \psi_l$ refers to the two leads (depicted blue in Fig. 1(a)). The system-bath coupling is mediated via the tunneling Hamiltonian

$$H_{SB} = \sum_{l,r} \sum_{n=1,2} (\gamma_0 + \gamma_x \hat{x}_n) \psi_l^\dagger \psi_r + \text{H.c.}, \quad (3)$$

where for small oscillation amplitudes we approximate the tunneling matrix elements as linear in the oscillator displacement, and for the sake of simplicity assume all tunneling amplitudes to be real and take the coupling to be symmetric. In order to treat non-Markovian effects of the fermionic reservoirs properly, we need to include an energy dependent density of states in the leads. This is done with a Lorentzian shaped density of states which means that an electron with energy ϵ_l in the left lead can

tunnel into states of the right lead with energy ε_r , broadened by L_c [24, 31–33]. If we compared Setup (A) with a scenario where the right reservoir is cut into two halves and therefore each oscillator is coupled to independent reservoirs on the right hand side, there would be no possibility to have any coherent effective coupling between the two oscillators. Hence, the common right reservoir is crucial for the generation of entanglement in this setup. The equation of motion for the reduced density matrix of the system up to second order in the tunneling can be written as (details are given in Ref. [34])

$$\begin{aligned} \dot{\rho}_S(t) = & -i \left[H_S + i\mathcal{K}_-^{(2)}(t)(\hat{x}_1 + \hat{x}_2)^2, \rho_S(t) \right] \\ & - \mathcal{K}_+^{(1)}(t) [\hat{x}_1 + \hat{x}_2, [\hat{x}_1 + \hat{x}_2, \rho_S(t)]] \\ & + \tilde{\mathcal{K}}_+^{(1)}(t) [\hat{x}_1 + \hat{x}_2, [\hat{p}_1 + \hat{p}_2, \rho_S(t)]] \\ & + \tilde{\mathcal{K}}_-^{(2)}(t) [\hat{x}_1 + \hat{x}_2, \{\hat{p}_1 + \hat{p}_2, \rho_S(t)\}] . \quad (4) \end{aligned}$$

We now investigate the system's dynamics, where we take the vacuum state as an initial state, which is Gaussian, allowing us to use the logarithmic negativity as a measure of entanglement. Here, we only discuss the case of zero electronic temperature. We show in Fig. 2 the logarithmic negativity for $\gamma_x = 0.1\sqrt{m\Omega}$ and its dependence on L_c . We clearly see that the initially separable state becomes entangled right after the interactions have been suddenly switched on. Due to this quench, energy conservation can be violated for a short time, which is the cause for the entanglement. Experimentally, the switching in this tunneling setup can be achieved by gates controlling the tunnel coupling via a resonance. Physically this means, that the rise time of the gates has to be shorter than $1/\Omega$, the timescale of the oscillators. Typical rise times of electronic gates can be as short as 60 ps [35]. For an oscillator with a resonance frequency of ~ 500 MHz (which can be brought into its ground state at low temperatures) these rise times are sufficient to accomplish the sudden switching. In its subsequent evolution, the entanglement oscillates in time, before it slowly decays. We find that low bias voltages show an increased logarithmic negativity compared to high bias voltages (not shown in the figure).

Setup B.— The major drawback of Setup (A) is the assumption that the which-path information of the tunneling electrons is lost because the common right reservoir cannot spatially distinguish the electrons. To circumvent this assumption, we now introduce an alternative realization based on a superconducting reservoir in the center and two independent normal metal reservoirs which nonetheless is capable of generating entanglement. This realization, Setup (B), is of an Andreev entangler type. In its original sense, the Andreev entangler [25] relies on the splitting of a Cooper-pair which is a spin-singlet. There, the split Cooper-pair from an s -wave superconductor can coherently tunnel via two different

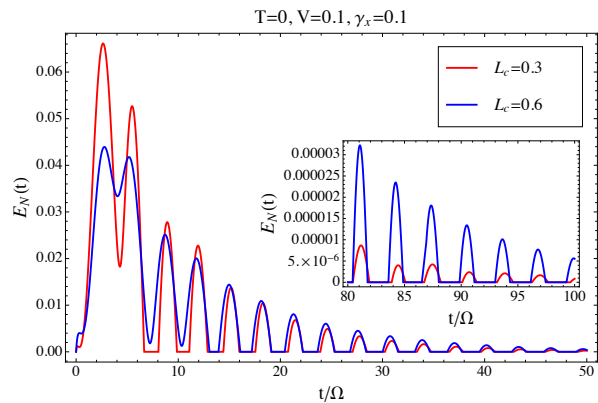


FIG. 2. (Color online) Logarithmic negativity for $L_c = 0.3\Omega$ (red) and $L_c = 0.6\Omega$ (blue). Here, the temperature $T = 0$, $V = 0.1\Omega$, and $\gamma_x = 0.1\sqrt{m\Omega}$. As can be seen, the qualitative dependence on the parameter L_c is rather small.

quantum dots into different leads. Throughout this process, the spin-singlet is preserved. Therefore, the Andreev entangler is capable of generating nonlocal spin-entangled electrons. In our context of NEMS, we use only charge properties of the split Cooper-pair as a mediator for an effective coupling between the oscillators. We show in Fig. 1(b) what such an Andreev entangler setup could look like. A superconducting island (orange) serves as a source of Cooper-pairs which can tunnel onto two different (conducting) nanomechanical oscillators. The process where the Cooper-pair is split and one electron tunnels to the lower oscillator and the other one tunnels to the upper oscillator gives rise to an effective coupling between them. The major improvements of Setup (B) over Setup (A) are: (i) the right common reservoir of Setup (A) is now cut into two halves; the resulting two normal metal leads are entirely *independent* (ii) both normal metal leads see the same tunneling processes; therefore their independence does not result in which-path information (iii) phase coherence lengths of superconductors are typically much longer than of normal metals which allows for a larger spatial extend of the center reservoir. Processes where one Cooper-pair tunnels to the upper/lower lead are not taken into account since they won't lead to any entanglement, but rather to a background tunneling current. As for instance described in Ref. [25], such processes are also energetically not favorable due to interaction effects in the leads and/or tunneling via a dot with a finite charging energy. Another possibility to favor the mentioned tunneling process, is to couple the superconductor to two Luttinger liquid leads as described in Ref. [36]. In NEMS this could be achieved by modeling the oscillators as one-dimensional nanowires or carbon nanotubes. We focus here on transport at energies in the superconducting gap Δ , which is the largest energy scale of the problem.

The system is again described by the Hamiltonian in Eq. (1). Now, the bath consists of three independent reservoirs, two normal metal leads, and a central superconducting region. The bath Hamiltonian becomes $H_B = H_I + H_U + H_D$ with $H_\alpha = \sum_{k\sigma} \varepsilon_{\alpha k} b_{\alpha k\sigma}^\dagger b_{\alpha k\sigma}$ ($\alpha = U, D$) and $H_I = \sum_{k\sigma} E_k \beta_{k\sigma}^\dagger \beta_{k\sigma}$. Here $E_k = \sqrt{(\varepsilon_k - \mu_S)^2 + \Delta^2}$ is the quasiparticle energy and $\beta_{k\sigma}$ is the quasiparticle annihilation operator in the superconductor. We take the superconductor to be grounded and each lead to be held at bias voltage V . The system-bath interaction is mediated by the tunneling of a split Cooper-pair. The process we focus on is therefore described by the effective tunneling Hamiltonian

$$H_{SB} = \sum_{k_1, k_2, k_3, k_4} T_U T_D b_{D-k_1\downarrow}^\dagger b_{Uk_2\uparrow}^\dagger c_{I-k_3\downarrow} c_{Ik_4\uparrow} \quad (5)$$

$$+ T_U T_D b_{Dk_1\uparrow}^\dagger b_{U-k_2\downarrow}^\dagger c_{I-k_3\downarrow} c_{Ik_4\uparrow} + \text{H.c.},$$

where $T_\alpha = \gamma_{0\alpha} + \gamma_{x\alpha} \hat{x}_\alpha$ and the electron annihilation operators for spin \uparrow and \downarrow are given by $c_{k\uparrow} = u_k \beta_{k\uparrow} + v_k \beta_{-k\downarrow}^\dagger$ and $c_{-k\downarrow} = u_k \beta_{-k\downarrow} - v_k \beta_{k\uparrow}^\dagger$, respectively. The effective Hamiltonian, cf. Eq. (5), exhibits two important but different contributions. The first contribution stems from terms of order $\gamma_0 \gamma_x$. These terms are qualitatively similar to terms of order γ_x in Setup (A), meaning that to second order in γ_x we expect a qualitatively similar behavior for the logarithmic negativity. Only considering these terms in the master equation, all kernels $\mathcal{K}(t)$ and $\tilde{\mathcal{K}}(t)$ for Setup (B) can be calculated in the same way as the kernels for Setup (A) (for details on the equation of motion for $\rho_S(t)$ for Setup (B) see Ref. [34]). In Fig. 3, we present the results for $E_N(t)$ for Setup (B). From this figure, we already see the similarity between Setup (A) and (B) and conclude that it is possible to entangle two nanomechanical resonators which are coupled to three independent reservoirs within an Andreev entangler setup. The second contribution of the Hamil-

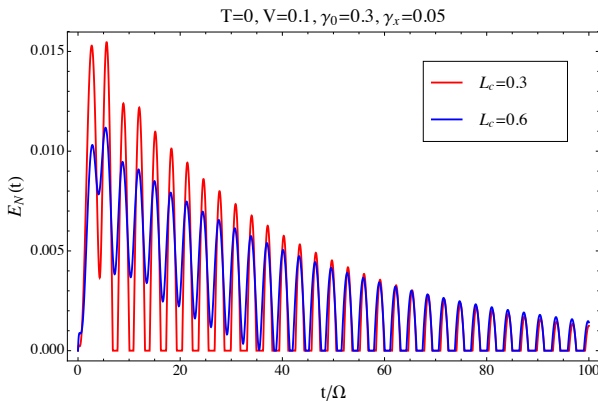


FIG. 3. (Color online) Logarithmic negativity for $L_c = 0.3$ (red) and $L_c = 0.6$ (blue). Here, the temperature $T = 0$, $V = 0.1 \Omega$, $\gamma_0 = 0.3$, and $\gamma_x = 0.05 \sqrt{m\Omega}$.

tonian Eq. (5) is due to terms of order γ_x^2 which give rise

to a direct coupling between the two oscillators, and are a new element in the Andreev entangler setup. Interestingly, these terms can even generate entanglement in the Markovian regime. In the low bias limit, we can apply the rotating wave approximation since the oscillators cannot be excited by the applied bias and excitations can only be swapped between them. The resulting master equation is of Lindblad form and given by

$$\dot{\rho}_S(t) = -i [H_S, \rho_S(t)] \quad (6)$$

$$+ \frac{\tilde{\rho} \pi^3 \gamma_x^4 V}{(m\Omega)^2} \left[A \rho_S(t) A - \frac{1}{2} \{A A, \rho_S(t)\} \right],$$

where the Lindblad operator $A = \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger$, and $\tilde{\rho} = \rho_U \rho_D \rho_S \rho_S$ with ρ_α and ρ_S being constant density of states in lead α and the superconductor, respectively. We also introduced bosonic annihilation $\hat{a}_i = (\hat{x}_i \sqrt{m\Omega} + i \hat{p}_i / \sqrt{m\Omega}) / \sqrt{2}$ and creation operators $\hat{a}_i^\dagger = (\hat{x}_i \sqrt{m\Omega} - i \hat{p}_i / \sqrt{m\Omega}) / \sqrt{2}$. To exemplify the dynamics generated by the Lindblad operator A , we choose $|n_1 = 1, n_2 = 1\rangle$ as an initial state (n_i being the occupation number of oscillator i). Due to Eq. (6) the dynamics will only generate states $|n_1 = 2, n_2 = 0\rangle$ and $|n_1 = 0, n_2 = 2\rangle$. The system will stay in this three-dimensional Hilbert space. The initial state evolves to $p_1(t) |1, 1\rangle + p_2(t) (|2, 0\rangle + |0, 2\rangle)$. A definite indicator for entanglement in this case is the prefactor $p_2(t)$ in front of the Bell state. Here, we just want to make the point that within the Andreev entangler setup we can generate an entangled state even in the Markovian regime.

Conclusion.— We introduced two setups, where we have shown that it is possible to generate entanglement between two spatially separated nanoelectromechanical oscillators, even if these two oscillators are not directly coupled to each other. In Setup (A), the two oscillators are indirectly coupled via two common fermionic baths which should have a rather long coherence time. With Setup (B), we introduced an entirely new way of entangling two nanomechanical oscillators in an electric setup based on the working principle of an Andreev entangler. Here, the coherent process where a Cooper-pair is split leads to an effective coupling of the two nanomechanical oscillators. For both setups, the generated entanglement persists over many oscillator periods, before the system becomes separable again.

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Supplementary Material for “Entanglement of nanoelectromechanical oscillators by Cooper-pair tunneling”

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I. DAMPING AND DECOHERENCE KERNELS OF SETUP (A)

Here, we give details on the equation of motion for $\rho_S(t)$ and the appearing kernels for Setup (A). We first simplify the system-bath coupling Hamiltonian and write

$$H_{SB} = \sum_{n=1,2} S_n E + S_n^\dagger E^\dagger, \quad (\text{S.1})$$

where, S_n and E denote arbitrary operators acting only on the Hilbert space of system and bath, respectively. We define the system and bath operators as $S_n = \gamma_0 + \gamma_x \hat{x}_n$ and $E = \psi_l^\dagger \psi_r$, respectively. With this, the equation of motion for the reduced density matrix of the system up to second order in the tunneling, Eq. (2) of the main text, can be written as Eq. (4) of the main text, where we defined the time dependent damping and decoherence kernels

$$\begin{aligned} \mathcal{K}_+^{(1)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \gamma_x^2 \cos(\Omega\tau), \\ \tilde{\mathcal{K}}_+^{(1)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \frac{\gamma_x^2}{m\Omega} \sin(\Omega\tau), \\ \mathcal{K}_-^{(2)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \gamma_x^2 \cos(\Omega\tau), \\ \tilde{\mathcal{K}}_-^{(2)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \frac{\gamma_x^2}{m\Omega} \sin(\Omega\tau) \end{aligned}$$

with

$$K^{(1)}(t) = \frac{1}{2} \langle \{E(t), E^\dagger(0)\} \rangle, \quad (\text{S.2})$$

$$K^{(2)}(t) = \frac{1}{2} \langle [E(t), E^\dagger(0)] \rangle. \quad (\text{S.3})$$

We see from the equation of motion for $\rho_S(t)$ that its time evolution is governed by the time dependent damping and diffusion kernels $\mathcal{K}_{+/-}^{1/2}(t)$ and $\tilde{\mathcal{K}}_{+/-}^{1/2}(t)$. We will briefly sketch the calculation of the kernels $K^{1/2}(t)$. Due to the fact that the equation of motion for $\rho_S(t)$ is of second order in H_{SB} , the only possibility of including non-Markovian effects is by considering an energy dependent density of states in the leads. Including non-Markovian effects leads to a finite correlation time in the leads, and is a key ingredient for the entanglement. Non-Markovian effects in the reservoirs and their influence on entanglement of two quantum systems have also been studied in Refs. [1–4]. The kernels $K^{(m)}(t)$ are given by

$$K^{(m)}(t) = \frac{1}{2} \int d\varepsilon_l \int d\varepsilon_r J(\varepsilon_l, \varepsilon_r) e^{i(\varepsilon_l - \varepsilon_r)t} \left[n(\varepsilon_l)[1 - n(\varepsilon_r)] - (-1)^m n(\varepsilon_r)[1 - n(\varepsilon_l)] \right],$$

where $n(\varepsilon_x) = [e^{\beta(\varepsilon_x - \mu_x)} + 1]^{-1}$ is the Fermi distribution function and we introduce an energy dependent spectral function

$$J(\varepsilon_l, \varepsilon_r) = \sum_{k,q} \delta(\varepsilon_l - \varepsilon_k) \delta(\varepsilon_r - \varepsilon_q).$$

To account for a finite lifetime of quasiparticles in the leads, the δ -functions are smeared out and replaced by Lorentzians of width L_c

$$J(\varepsilon_l, \varepsilon_r) = \sum_{k,q} \frac{L_c}{(\varepsilon_l - \varepsilon_k)^2 + L_c^2} \frac{L_c}{(\varepsilon_r - \varepsilon_q)^2 + L_c^2}. \quad (\text{S.4})$$

The largest contribution of each of the independent sums in Eq. (S.4) will come from the energies close to the Fermi level of each lead. We further restrict ourselves to the regime of low applied bias voltages ($V < L_c$). This is just a simplification in order to keep the number of parameters in the problem as low as possible. With these assumptions, the energy dependent spectral function can be written as

$$J(\varepsilon_l, \varepsilon_r) = \frac{1}{(\varepsilon_l - \varepsilon_r)^2 + L_c^2}. \quad (\text{S.5})$$

Physically, Eq. (S.5) means that an electron with energy ε_l in the left lead can tunnel into states of the right lead with energy ε_r , broadened by L_c [1, 5–7]. With the spectral function in Eq. (S.5), the time dependent kernels $\mathcal{K}_{+/-}^{1/2}(t)$ and $\tilde{\mathcal{K}}_{+/-}^{1/2}(t)$ can now be calculated analytically, but the resulting expressions are too lengthy to be stated here.

II. EQUATION OF MOTION AND TIME DEPENDENT KERNELS OF SETUP (B)

The equation of motion and the calculation of the kernels for the master equation in the case of Setup (B) are very similar to Setup (A). Therefore we will only briefly sketch the main ideas.

The quasiparticle operators of the Hamiltonian H_I are related to electron annihilation operators through the Bogoliubov transformation

$$\begin{aligned} c_{k\uparrow} &= u_k \beta_{k\uparrow} + v_k \beta_{-k\downarrow}^\dagger, \\ c_{-k\downarrow} &= u_k \beta_{-k\downarrow} - v_k \beta_{k\uparrow}^\dagger, \end{aligned}$$

where $u_k = \sqrt{1/2 + \xi_k/(2E_k)}$ and $v_k = \sqrt{1/2 - \xi_k/(2E_k)}$ with $\xi_k = \varepsilon_k - \mu_S$. As before, we introduce system and bath operators

$$\begin{aligned} S_0 &= \gamma_{0U} \gamma_{0D}, \\ S_1 &= \gamma_{0D} \gamma_{xU} \hat{x}_U, \\ S_2 &= \gamma_{0U} \gamma_{xD} \hat{x}_D, \\ S_3 &= \gamma_{xU} \gamma_{xD} \hat{x}_D \hat{x}_U, \end{aligned}$$

and

$$E = b_{D-k_1\downarrow}^\dagger b_{Uk_2\uparrow}^\dagger c_{I-k_3\downarrow} c_{Ik_4\uparrow}, \quad (\text{S.6})$$

respectively, which allow us to write the system-bath coupling Hamiltonian in a similar way as in Eq. (S.1). With this, the equation of motion for Setup (B) is similar to Eq. (4) of the main text and can be written as

$$\begin{aligned} \dot{\rho}_S(t) = & -i \left[H_S + i\mathcal{K}_-^{(2)}(t)(\hat{x}_1 + \hat{x}_2)^2, \rho_S(t) \right] \\ & - \mathcal{K}_+^{(1)}(t) [\hat{x}_1 + \hat{x}_2, [\hat{x}_1 + \hat{x}_2, \rho_S(t)]] \\ & + \tilde{\mathcal{K}}_+^{(1)}(t) [\hat{x}_1 + \hat{x}_2, [\hat{p}_1 + \hat{p}_2, \rho_S(t)]] \\ & + \tilde{\mathcal{K}}_-^{(2)}(t) [\hat{x}_1 + \hat{x}_2, \{\hat{p}_1 + \hat{p}_2, \rho_S(t)\}] \\ & - \mathcal{K}_+^{(3,1)}(t) [\hat{x}_1 \hat{x}_2, [\hat{x}_1 \hat{x}_2, \rho_S(t)]] \\ & - \mathcal{K}_+^{(3,2)}(t) [\hat{x}_1 \hat{x}_2, [\hat{p}_1 \hat{p}_2, \rho_S(t)]] \\ & + \mathcal{K}_+^{(3,3)}(t) [\hat{x}_1 \hat{x}_2, [\hat{x}_1 \hat{p}_2 + \hat{x}_2 \hat{p}_1, \rho_S(t)]] \\ & - \mathcal{K}_-^{(3,1)}(t) [\hat{x}_1 \hat{x}_2, \{\hat{x}_1 \hat{x}_2, \rho_S(t)\}] \\ & - \mathcal{K}_-^{(3,2)}(t) [\hat{x}_1 \hat{x}_2, \{\hat{p}_1 \hat{p}_2, \rho_S(t)\}] \\ & + \mathcal{K}_-^{(3,3)}(t) [\hat{x}_1 \hat{x}_2, \{\hat{x}_1 \hat{p}_2 + \hat{x}_2 \hat{p}_1, \rho_S(t)\}], \end{aligned} \quad (\text{S.7})$$

where we have defined the time dependent memory kernels in Eq. (S.7) as

$$\begin{aligned}
\mathcal{K}_+^{(1)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \gamma_0^2 \gamma_x^2 \cos(\Omega\tau), \\
\tilde{\mathcal{K}}_+^{(1)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \frac{\gamma_0^2 \gamma_x^2}{m\Omega} \sin(\Omega\tau), \\
\mathcal{K}_-^{(2)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \gamma_0^2 \gamma_x^2 \cos(\Omega\tau), \\
\tilde{\mathcal{K}}_-^{(2)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \frac{\gamma_0^2 \gamma_x^2}{m\Omega} \sin(\Omega\tau), \\
\mathcal{K}_+^{(3,1)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \gamma_x^4 \cos(\Omega\tau) \cos(\Omega\tau), \\
\mathcal{K}_+^{(3,2)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \frac{\gamma_x^4}{m^2 \Omega^2} \sin(\Omega\tau) \sin(\Omega\tau), \\
\mathcal{K}_+^{(3,3)}(t) &= \int_0^t d\tau \left[K^{(1)}(\tau) + K^{(1)}(-\tau) \right] \frac{\gamma_x^4}{m\Omega} \cos(\Omega\tau) \sin(\Omega\tau), \\
\mathcal{K}_-^{(3,1)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \gamma_x^4 \cos(\Omega\tau) \cos(\Omega\tau), \\
\mathcal{K}_-^{(3,2)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \frac{\gamma_x^4}{m^2 \Omega^2} \sin(\Omega\tau) \sin(\Omega\tau), \\
\mathcal{K}_-^{(3,3)}(t) &= \int_0^t d\tau \left[K^{(2)}(\tau) - K^{(2)}(-\tau) \right] \frac{\gamma_x^4}{m\Omega} \cos(\Omega\tau) \sin(\Omega\tau).
\end{aligned}$$

The kernels $K^{(1/2)}(t)$ are given in Eq. (S.2) and (S.3) with E given in Eq. (S.6). The actual calculation of the kernels for the Andreev entangler setup goes along the same lines as for setup (A).

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